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An intuitionistic fuzzy Dombi–Archimedean RANCOM–AROMAN framework for AI-enabled telemedicine platform selection

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Abstract

Artificial intelligence (AI)-enabled telemedicine platform selection involves multiple conflicting criteria and uncertain expert judgments. Conventional multi-criteria decision-making (MCDM) approaches often fail to preserve membership, non-membership, and hesitation throughout the complete decision process. Existing intuitionistic fuzzy Dombi models lack fully integrated aggregation, criterion weighting, dual-normalization fusion, and ranking mechanisms with rigorous endpoint treatment. This study develops a mathematically consistent and robust intuitionistic fuzzy Dombi–Archimedean (IFDA) decision framework that integrates IFDA–RANCOM–AROMAN to manage uncertain evaluations from criterion weighting to final ranking. IFDA–RANCOM determines criterion weights, whereas IFDA–AROMAN performs dual normalization, fuzzy fusion, benefit–cost utility construction, and alternative ranking. The framework is illustrated through the selection of six AI-enabled telemedicine platforms for rural healthcare. The obtained weights are $\omega = (0.36, 0.28, 0.20, 0.12, 0.04)$, and the baseline ranking is $(A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_6 \succ A_4)$. RANCOM–AROMAN reproduces the same complete ranking, while other methods confirm a similar leading group. The leading alternatives remain stable under parameter variations and ($\pm 15\%$) criterion-weight perturbations. The framework provides a reproducible uncertainty-preserving decision mechanism. The proposed model offers a rigorous and robust tool for complex fuzzy decision-making applications.

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1. Introduction

Healthcare technology selection is seldom governed by a single performance indicator. A telemedicine platform may deliver strong diagnostic decision-support performance while also requiring substantial deployment cost, specialized staff training, or difficult integration with electronic health record systems. In contrast, a low-cost platform may appear suitable for rural clinics but may fail to meet expectations concerning privacy, security, interoperability, and responsible governance. Such conflicts make the selection problem intrinsically multi-criteria and uncertain. A suitable decision model must therefore represent supportive evidence, opposing evidence, and incomplete expert judgement within the same mathematical structure. Fuzzy set theory, introduced by Zadeh [1], provides the basic language for gradual membership. Classical fuzzy sets, however, assign only a membership degree and do not explicitly distinguish rejection from hesitation. Atanassov's intuitionistic fuzzy sets (IFSs) overcome this limitation by assigning both membership and non-membership degrees subject to $0 \leq \mu + \nu \leq 1$ [2, 3]. The remaining quantity, $\pi = 1 - \mu - \nu$, gives a direct measure of hesitation or indeterminacy, which is particularly relevant when expert assessments are based on partial or imperfect information. The distance between IFNs, as formalized by Szmidt and Kacprzyk [4], further enables robust comparison and ranking under uncertainty.

Intuitionistic fuzzy multi-criteria decision-making (MCDM) has become an important research area because it combines uncertainty representation with practical ranking mechanisms. Yager's ordered weighted averaging operator introduced position-based aggregation [5], while Xu and Yager [6] extended geometric aggregation to the intuitionistic fuzzy setting, and Xu [7] developed a broad class of IF aggregation operators supported by score and accuracy functions. Subsequent studies incorporated Einstein operations to obtain more adaptable aggregation mechanisms [8, 9].

The Dombi family of triangular norms and conorms, introduced by Dombi [10], is particularly useful because it contains a positive flexibility parameter that regulates aggregation strictness. Archimedean t-norm theory, as consolidated by Klement *et al.* [11] and further discussed by Grabisch *et al.* [12], adds a generator-based foundation for constructing continuous and associative fuzzy operations. Recently, Liu *et al.* [13] and Seikh and Mandal [14] applied Dombi operators to intuitionistic fuzzy environments, demonstrating that the final ranking is shaped not only by the decision matrix and criterion weights but also by the compensation behaviour embedded in the aggregation operator. For this reason, a rigorous MCDM framework should not merely state an operator; it should define its operational laws, prove its mathematical properties, show the transformation of raw data, and evaluate the stability of the ranking.

Classical MCDM frameworks such as the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), the Analytical Hierarchy Process (AHP), entropy weighting, and the Criteria Importance Through Intercriteria Correlation (CRITIC) weighting method remain widely used in decision science [15–19]. IFSs have been successfully integrated into these methods for personnel selection, supplier evaluation, and medical decision support [20–22]. Recent advances include modified intuitionistic fuzzy distances by Ejegwa *et al.* [23] and extensions of hesitant fuzzy VIKOR by Raza et al. [24].

The application studied here concerns the selection of an artificial intelligence (AI)-enabled telemedicine platform for rural healthcare deployment. This problem is practically significant because modern healthcare systems increasingly depend on remote consultation, decision-support tools, interoperable data exchange, and privacy-preserving analytics. The World Health Organization's global digital health strategy stresses coordinated digital health adoption as a means of strengthening health systems [25], while WHO guidance on artificial intelligence in health further emphasizes ethical governance, safety, transparency, and accountability [26]. Related governance instruments, including the National Institute of Standards and Technology (NIST) Cybersecurity Framework 2.0, provide a practical structure for cyber-risk governance in healthcare technology [27]. These technical, ethical, and operational requirements make telemedicine platform selection a relevant real-world decision problem under uncertainty.

The construction of flexible aggregation operators for IF environments rests on the foundational theory of triangular norms and statistical metric spaces [28], while the reliability of subjective criterion weights is often ensured through consistency checks such as those employed in the Analytical Hierarchy Process [29]. To quantify the uncertainty inherent in such evaluations, entropy measures for intuitionistic fuzzy sets [30] provide a robust tool for assessing fuzziness and hesitation prior to aggregation. Building on these fundamentals, Liu and Wang [31] and Ye [32] developed comprehensive IF MCDM frameworks and weighted averaging mechanisms that accommodate both membership and non-membership degrees. Recently, the integration of Dombi operations has enabled parameterised control over compensation behaviour in fuzzy aggregation [33], yet a unified reconstruction under pure Archimedean generators and the strict intuitionistic condition $\mu + \nu \leq 1$ remains unexplored, which forms the central motivation of this work.

Recent cybersecurity studies have applied machine-learning and uncertainty-based approaches to malware classification, anomaly detection, and network-intrusion identification, demonstrating the importance of systematic and data-informed security assessment [34–36]. Related fuzzy decision-making investigations have further shown that aggregation-based models can support the evaluation of security alternatives when expert judgments are imprecise, incomplete, or conflicting [37–40]. These studies motivate the development of a transparent and adjustable IF framework for ranking network-security systems without attributing cybersecurity claims to unrelated mathematical literature.

The mathematical reliability of decision-support models can also benefit from developments in operator theory, numerical approximation, dynamical-system stability, inequality analysis, and generalized uncertainty structures [41–45]. Although these studies are not directly concerned with cybersecurity, they provide broader methodological perspectives on stability, computation, control, and uncertainty representation that are relevant to the theoretical development of aggregation-based decision models. Accordingly, they are cited here as supporting mathematical literature rather than as evidence of cybersecurity standards, intrusion-detection methods, or security-control practices.

Criterion weighting and alternative ranking are distinct stages of an MCDM model. Ranking Comparison (RANCOM) derives subjective criterion weights from an ordered ranking-comparison structure and reduces the number and complexity of judgments required by dense pairwise-comparison techniques [46]. Its fuzzy extension permits uncertain criterion relations [47]. In the present framework, experts first provide criterion-importance IFNs. These IFNs are aggregated by the proposed Dombi–Archimedean averaging operator, converted to bounded score values, ordered by the score–accuracy rule, and then used to construct the RANCOM relation matrix. Thus, uncertainty is preserved until the criterion order has been established.

AROMAN, the Alternative Ranking Order Method Accounting for Two-Step Normalization, combines linear and vector normalization and separates benefit and cost effects in the final appraisal [48]. Its two-normalization structure reduces dependence on a single scaling rule. Fuzzy and intuitionistic fuzzy AROMAN variants have been applied to digital-economy assessment, EcoPort evaluation, wastewater-treatment technology selection, and logistics-provider selection [49–52]. Nevertheless, the two normalized matrices are commonly fused by a crisp arithmetic operation, after which the proposed fuzzy algebra has little influence on the remaining stages. This limits the methodological role of the aggregation mechanism and makes it difficult to determine how nonlinear compensation affects the final order.

The present study addresses this limitation by integrating the same intuitionistic fuzzy Dombi–Archimedean (IFDA) mechanism throughout the RANCOM–AROMAN chain. The IFDA averaging operator aggregates criterion-importance assessments, aggregates multiple-expert alternative evaluations, fuses the linear and vector normalized matrices, and constructs separate benefit and cost-desirability utility IFNs. Cost criteria are reversed once, after normalization fusion, through the IF complement. This avoids repeated or contradictory cost transformations. The final AROMAN appraisal is then calculated from bounded scalar utilities derived from the IF benefit and cost-desirability results.

1.1. Motivation and research gap

The study is motivated by the following connected gaps. Existing operator-based studies frequently apply a newly defined operator only at the initial aggregation stage and then use an unrelated crisp ranking method.

Endpoint conventions for Dombi generators are often omitted, preventing exact reproduction when a membership or non-membership component equals 0 or 1.

Some weighted, ordered, and hybrid formulations do not distinguish attribute weights from position weights or do not impose the normalization conditions required by idempotency.

Ordered-operator monotonicity is often stated without considering that score-based sorting permutations may change after the inputs are transformed.

Existing IF-AROMAN formulations do not provide a direct IFDA fusion of the linear and vector normalized matrices together with closure proofs and exact implementable equations.

The role of the Dombi parameter is sometimes described as neutral or central without mathematical justification, even though the preferred alternative may change with that parameter.

These gaps lead to four research questions: how can endpoint-complete Dombi operations be defined on IFNs; how can weighted, ordered, and hybrid averaging/geometric operators be made closed and notation-consistent; how can IFDA aggregation be incorporated separately into RANCOM weighting and AROMAN ranking; and how sensitive is the resulting telemedicine-platform ranking to the parameters and criterion weights?

The main contributions of this study are summarized as follows: A unified IFDA–RANCOM–AROMAN framework is developed for multi-criteria decision-making under intuitionistic fuzzy information, where uncertainty is preserved from expert assessment to final alternative ranking.

An IFDA–RANCOM weighting mechanism is proposed to aggregate uncertain criterion-importance evaluations, determine criterion priorities through the score–accuracy rule, construct the RANCOM relation matrix, and obtain a normalized criterion-weight vector.

An IFDA–AROMAN ranking procedure is formulated by integrating linear and vector normalization, intuitionistic fuzzy embedding, multi-expert aggregation, Dombi–Archimedean normalization fusion, cost complementation, benefit–cost utility construction, and final appraisal.

In contrast to conventional approaches that employ fuzzy aggregation only at an initial stage, the proposed model applies the IFDA mechanism consistently throughout the principal uncertainty-sensitive stages of the RANCOM–AROMAN decision process.

The mathematical foundation of the framework is established through endpoint-complete Dombi–Archimedean operations and valid intuitionistic fuzzy aggregation formulations satisfying closure and essential aggregation properties.

The practical applicability of the proposed framework is demonstrated through the selection of six AI-enabled telemedicine platforms under five benefit and cost criteria relevant to rural healthcare.

Sensitivity analysis of criterion-weight perturbation and comparative evaluation confirm the stability of the leading alternatives and demonstrate the robustness and reliability of the proposed IFDA–RANCOM–AROMAN framework.

Section 2 fixes the IF notation, ordering rule, strict Archimedean generators, endpoint-complete Dombi functions, and the basic RANCOM and AROMAN concepts. Section 3 defines the IFDA operational laws and proves their closure and algebraic identities. Section 4 develops the six weighted, ordered, and hybrid averaging/geometric operators, including stepwise closed-form derivations. Section 5 presents the proposed IFDA–RANCOM and IFDA–AROMAN methods as separate mathematical procedures. Section 6 reports the telemedicine-platform case study. Section 7 retains the parameter-sensitivity, weight-perturbation, and comparative analyses. Section 8 concludes the paper.

2. Preliminaries

Table 1 is the authoritative notation table. No symbol outside this table is used without an explicit local definition.

Definition 2.1 (Intuitionistic fuzzy set and IFN)**.** Let X be a universe. An intuitionistic fuzzy set [2] is

$$\mathcal{I} = \{\langle x, \mu_{\mathcal{I}}(x), \nu_{\mathcal{I}}(x)\rangle : x \in X\},$$

where $\mu_{\mathcal{I}}, \nu_{\mathcal{I}} : X \to [0,1]$ and $\mu_{\mathcal{I}}(x) + \nu_{\mathcal{I}}(x) \le 1$. For a fixed x, $\gamma = (\mu, \nu)$ is an IFN and $\pi = 1 - \mu - \nu$.

Definition 2.2 (IF order and ranking rule)**.** For IFNs $\gamma_1 = (\mu_1, \nu_1)$ and $\gamma_2 = (\mu_2, \nu_2)$,

$$\gamma_1 \preceq_{IF} \gamma_2 \quad\Longleftrightarrow\quad \mu_1 \le \mu_2 \text{ and } \nu_1 \ge \nu_2.$$

For total ranking, define $\mathscr{S}(\gamma) = \mu - \nu$ and $\mathscr{H}(\gamma) = \mu + \nu$. Rank first by decreasing score and, when scores are equal, by decreasing accuracy. Equal score and accuracy determine the same pair (μ, ν); identical IFNs are tied and their original indices are used only to make the sorting permutation deterministic.

Definition 2.3 (Strict Archimedean t-norm generator)**.** A strict Archimedean t-norm generator [12] is a continuous, strictly decreasing bijection

$$g : [0,1] \longrightarrow [0,\infty]$$

with $g(1) = 0$ and $g(0) = \infty$. Its extended inverse $g^{-1} : [0,\infty] \to [0,1]$ satisfies $g^{-1}(0) = 1$ and $g^{-1}(\infty) = 0$. The associated t-norm is

$$T(x,y) = g^{-1}(g(x) + g(y)), \qquad x, y \in [0,1],$$

where addition is in the extended nonnegative reals.

Definition 2.4 (Strict Archimedean t-conorm generator)**.** A strict Archimedean t-conorm generator [12] is a continuous, strictly increasing bijection

$$f : [0,1] \longrightarrow [0,\infty]$$

with $f(0) = 0$ and $f(1) = \infty$. Its extended inverse satisfies $f^{-1}(0) = 0$ and $f^{-1}(\infty) = 1$. The associated t-conorm is

$$S(x,y) = f^{-1}(f(x) + f(y)).$$

When $f(x) = g(1-x)$, S and T are dual: $S(x,y) = 1 - T(1-x, 1-y)$.

Definition 2.5 (Dombi generators and inverses)**.** For the single parameter $\xi > 0$, following Dombi [10], define on the extended unit interval

$$L_\xi(x) = \left(\frac{x}{1-x}\right)^{\xi}, \qquad L_\xi(0) = 0, \qquad L_\xi(1) = \infty,$$
$$G_\xi(x) = \left(\frac{1-x}{x}\right)^{\xi}, \qquad G_\xi(0) = \infty, \qquad G_\xi(1) = 0. \tag{1}$$

Their inverses for $z \in [0,\infty]$ are

$$s_\xi(z) = \frac{z^{1/\xi}}{1 + z^{1/\xi}}, \qquad t_\xi(z) = \frac{1}{1 + z^{1/\xi}}, \tag{2}$$

with $s_\xi(0) = 0$, $s_\xi(\infty) = 1$, $t_\xi(0) = 1$, and $t_\xi(\infty) = 0$. Moreover, $G_\xi(x) = L_\xi(1-x)$ and $t_\xi(z) = 1 - s_\xi(z)$.

Definition 2.6 (Dombi t-norm and t-conorm)**.** The Dombi t-conorm and t-norm [10] are

$$S_\xi(x,y) = s_\xi(L_\xi(x) + L_\xi(y)) = \frac{\left[L_\xi(x) + L_\xi(y)\right]^{1/\xi}}{1 + \left[L_\xi(x) + L_\xi(y)\right]^{1/\xi}},$$
$$T_\xi(x,y) = t_\xi(G_\xi(x) + G_\xi(y)) = \frac{1}{1 + \left[G_\xi(x) + G_\xi(y)\right]^{1/\xi}}. \tag{3}$$

The formulas use the endpoint conventions of Definition 2.5; consequently $S_\xi(0,x) = x$, $S_\xi(1,x) = 1$, $T_\xi(1,x) = x$, and $T_\xi(0,x) = 0$.

2.1. RANCOM and AROMAN preliminaries

Let $\overline{C}_j$, $j = 1, \ldots, n$, denote criteria. RANCOM uses an ordered criterion structure and defines

$$r_{jk} = \begin{cases} 1, & \overline{C}_j \succ \overline{C}_k, \\ \frac{1}{2}, & \overline{C}_j \sim \overline{C}_k, \\ 0, & \overline{C}_j \prec \overline{C}_k, \end{cases} \qquad s_j = \sum_{k=1}^{n} r_{jk}, \qquad \omega_j = \frac{s_j}{\sum_{\ell=1}^{n} s_\ell}. \tag{4}$$

For a crisp matrix $X = [x_{ij}]$, AROMAN uses two normalizations, here written as

$$\ell_{ij} = \frac{x_{ij} - x_j^-}{x_j^+ - x_j^-}, \qquad v_{ij} = \frac{x_{ij}}{\sqrt{\sum_{r=1}^{m} x_{rj}^2}}, \tag{5}$$

where $x_j^- = \min_i x_{ij}$ and $x_j^+ = \max_i x_{ij}$.

3. Dombi–Archimedean operations on IFNs

The mathematical development uses only the two-component IFN $\gamma = (\mu, \nu)$ established in Definition 2.1.

Definition 3.1 (IF Dombi operations)**.** For IFNs $\gamma_1 = (\mu_1, \nu_1)$, $\gamma_2 = (\mu_2, \nu_2)$, $\lambda \ge 0$, and $\xi > 0$, define

$$\gamma_1 \oplus_{\mathcal{AD}} \gamma_2 = (S_\xi(\mu_1, \mu_2),\ T_\xi(\nu_1, \nu_2)), \tag{6}$$
$$\gamma_1 \otimes_{\mathcal{AD}} \gamma_2 = (T_\xi(\mu_1, \mu_2),\ S_\xi(\nu_1, \nu_2)), \tag{7}$$
$$\lambda \odot_{\mathcal{AD}} \gamma_1 = (s_\xi(\lambda L_\xi(\mu_1)),\ t_\xi(\lambda G_\xi(\nu_1))), \tag{8}$$
$$\gamma_1^{\circledast_{\mathcal{AD}} \lambda} = (t_\xi(\lambda G_\xi(\mu_1)),\ s_\xi(\lambda L_\xi(\nu_1))). \tag{9}$$

Thus $0 \odot_{\mathcal{AD}} \gamma = (0,1)$ and $\gamma^{\circledast_{\mathcal{AD}} 0} = (1,0)$.

Table 1. Notation used throughout the revised manuscript.

Symbol	Meaning
$\gamma = (\mu, \nu)$	intuitionistic fuzzy number (IFN)
$\pi = 1 - \mu - \nu$	hesitation degree
$\mathscr{S}(\gamma) = \mu - \nu$	score function
$\mathscr{H}(\gamma) = \mu + \nu$	accuracy function
$\preceq_{IF}$	IF partial order
g, g^{-1}	strict Archimedean t-norm generator and inverse
f, f^{-1}	strict Archimedean t-conorm generator and inverse
L_ξ, G_ξ	Dombi increasing/decreasing generators
s_ξ, t_ξ	inverses of L_ξ, G_ξ
S_ξ, T_ξ	Dombi t-conorm and t-norm
$\xi > 0$	unique Dombi flexibility parameter
ω_j	normalized aggregation or position weight
Ω_j	normalized attribute weight in a hybrid operator
$\eta \in [0, 1]$	hybrid balancing factor
σ	sorting permutation

Theorem 3.1 (Closure)**.** *Every result in Equations (6)–(9) is an IFN.*

Proof. Because $\mu_i + \nu_i \le 1$, $\mu_i \le 1 - \nu_i$. Since L_ξ is increasing and $G_\xi(\nu_i) = L_\xi(1-\nu_i)$, $L_\xi(\mu_i) \le G_\xi(\nu_i)$. Summing and applying increasing s_ξ gives

$$s_\xi\left(\sum_i L_\xi(\mu_i)\right) \le s_\xi\left(\sum_i G_\xi(\nu_i)\right) = 1 - t_\xi\left(\sum_i G_\xi(\nu_i)\right),$$

which proves closure of $\oplus_{\mathcal{AD}}$; multiplying both sides by $\lambda \ge 0$ proves closure of $\lambda \odot_{\mathcal{AD}} \gamma$. For $\otimes_{\mathcal{AD}}$, $1-\mu_i \ge \nu_i$ implies $G_\xi(\mu_i) = L_\xi(1-\mu_i) \ge L_\xi(\nu_i)$. Hence $t_\xi(\sum_i G_\xi(\mu_i)) + s_\xi(\sum_i L_\xi(\nu_i)) \le 1$. The same argument after multiplication by λ proves closure of the power operation. All components lie in $[0, 1]$ by Definition 2.5. □

Example 3.1. Let $\gamma_1 = (0.5, 0.2)$, $\gamma_2 = (0.3, 0.4)$, $\xi = 2$, and $\lambda = 0.3$. Direct substitution gives

$$\begin{aligned}
\gamma_1 \oplus_{\mathcal{AD}} \gamma_2 &= (0.5211, 0.1897), \quad 0.5211 + 0.1897 = 0.7108,\\
\gamma_1 \otimes_{\mathcal{AD}} \gamma_2 &= (0.2826, 0.4159), \quad 0.2826 + 0.4159 = 0.6985,\\
0.3 \odot_{\mathcal{AD}} \gamma_1 &= (0.3539, 0.3134), \quad 0.3539 + 0.3134 = 0.6673,\\
\gamma_1^{\circledast_{\mathcal{AD}} 0.3} &= (0.6461, 0.1204), \quad 0.6461 + 0.1204 = 0.7665.
\end{aligned}$$

All four outputs satisfy the IF feasibility condition.

Theorem 3.2. *For IFNs γ_1, γ_2, $\lambda, \lambda_1, \lambda_2 \ge 0$,*

$$\begin{aligned}
\gamma_1 \oplus_{\mathcal{AD}} \gamma_2 &= \gamma_2 \oplus_{\mathcal{AD}} \gamma_1,\\
\gamma_1 \otimes_{\mathcal{AD}} \gamma_2 &= \gamma_2 \otimes_{\mathcal{AD}} \gamma_1,\\
\lambda \odot_{\mathcal{AD}} (\gamma_1 \oplus_{\mathcal{AD}} \gamma_2) &= (\lambda \odot_{\mathcal{AD}} \gamma_1) \oplus_{\mathcal{AD}} (\lambda \odot_{\mathcal{AD}} \gamma_2),\\
(\lambda_1 + \lambda_2) \odot_{\mathcal{AD}} \gamma_1 &= (\lambda_1 \odot_{\mathcal{AD}} \gamma_1) \oplus_{\mathcal{AD}} (\lambda_2 \odot_{\mathcal{AD}} \gamma_1),\\
(\gamma_1 \otimes_{\mathcal{AD}} \gamma_2)^{\circledast_{\mathcal{AD}} \lambda} &= \gamma_1^{\circledast_{\mathcal{AD}} \lambda} \otimes_{\mathcal{AD}} \gamma_2^{\circledast_{\mathcal{AD}} \lambda},\\
\gamma_1^{\circledast_{\mathcal{AD}} (\lambda_1 + \lambda_2)} &= \gamma_1^{\circledast_{\mathcal{AD}} \lambda_1} \otimes_{\mathcal{AD}} \gamma_1^{\circledast_{\mathcal{AD}} \lambda_2}.
\end{aligned}$$

Proof. Commutativity follows from addition of generator values. For the third identity, applying L_ξ to the membership component of both sides gives $\lambda[L_\xi(\mu_1) + L_\xi(\mu_2)]$; applying G_ξ to the non-membership component gives $\lambda[G_\xi(\nu_1) + G_\xi(\nu_2)]$. Injectivity of s_ξ and t_ξ proves equality. The fourth identity follows by distributing $\lambda_1 + \lambda_2$. The final two identities follow identically from G_ξ on membership and L_ξ on non-membership. No undefined operands or duplicated cross-identities are used. □

4. Weighted, ordered, and hybrid IF Dombi operators

Throughout this section, every weight vector is nonnegative and normalized unless a stronger induction claim explicitly permits arbitrary nonnegative coefficients. Zero weights are interpreted through the endpoint conventions in Definition 2.5. Write $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_r)$ and $\boldsymbol{\gamma}' = (\gamma_1', \ldots, \gamma_r')$, with $\boldsymbol{\gamma} \preceq_{IF} \boldsymbol{\gamma}'$ meaning $\gamma_j \preceq_{IF} \gamma_j'$ for every j. For a collection $\gamma_j = (\mu_j, \nu_j)$, define the admissible componentwise IF bounds

$$\begin{aligned}
\gamma^- &= (\mu^-, \nu^-) = \left(\min_j \mu_j, \max_j \nu_j\right),\\
\gamma^+ &= (\mu^+, \nu^+) = \left(\max_j \mu_j, \min_j \nu_j\right).
\end{aligned} \tag{10}$$

4.1. IF Dombi–Archimedean weighted averaging operator

Definition 4.1 (IFDADWA)**.** For IFNs $\gamma_j = (\mu_j, \nu_j)$ and weights $\omega_j \ge 0$ satisfying $\sum_{j=1}^r \omega_j = 1$, define

$$\mathrm{IFDADWA}_\xi(\gamma_1, \ldots, \gamma_r; \boldsymbol{\omega}) = \bigoplus_{j=1}^{r} (\omega_j \odot_{\mathcal{AD}} \gamma_j). \tag{11}$$

Theorem 4.1 (Closed form of IFDADWA)**.**

$$\begin{aligned}
&\mathrm{IFDADWA}_\xi(\gamma_1, \ldots, \gamma_r; \boldsymbol{\omega})\\
&= \left(s_\xi\left(\sum_{j=1}^r \omega_j L_\xi(\mu_j)\right), t_\xi\left(\sum_{j=1}^r \omega_j G_\xi(\nu_j)\right)\right).
\end{aligned} \tag{12}$$

Proof. We prove the stronger identity for arbitrary coefficients $a_j \geq 0$:

$$\bigoplus_{j=1}^{r}(a_j \odot_{\mathcal{AD}} \gamma_j) = \left(s_\xi\left(\sum_{j=1}^{r} a_j L_\xi(\mu_j)\right), t_\xi\left(\sum_{j=1}^{r} a_j G_\xi(\nu_j)\right)\right). \quad (13)$$

Step 1: Base case. For $r = 1$, Definition 3.1 gives

$$a_1 \odot_{\mathcal{AD}} \gamma_1 = (s_\xi(a_1 L_\xi(\mu_1)), t_\xi(a_1 G_\xi(\nu_1))),$$

which is exactly (13).

Step 2: Induction hypothesis. Assume (13) holds for $r = q$. Put

$$A_q = \sum_{j=1}^{q} a_j L_\xi(\mu_j), \qquad B_q = \sum_{j=1}^{q} a_j G_\xi(\nu_j).$$

Then

$$\bigoplus_{j=1}^{q}(a_j \odot_{\mathcal{AD}} \gamma_j) = (s_\xi(A_q), t_\xi(B_q)).$$

Step 3: Add the $(q+1)$st term. By Definitions 3.1 and 2.6,

$$\left[\bigoplus_{j=1}^{q}(a_j \odot_{\mathcal{AD}} \gamma_j)\right] \oplus_{\mathcal{AD}} (a_{q+1} \odot_{\mathcal{AD}} \gamma_{q+1})$$
$$= \Big(S_\xi(s_\xi(A_q), s_\xi(a_{q+1} L_\xi(\mu_{q+1}))), T_\xi(t_\xi(B_q), t_\xi(a_{q+1} G_\xi(\nu_{q+1})))\Big).$$

Step 4: Apply the generator–inverse identities. Since $L_\xi(s_\xi(z)) = z$ and $G_\xi(t_\xi(z)) = z$,

$$S_\xi(s_\xi(A_q), s_\xi(a_{q+1} L_\xi(\mu_{q+1}))) = s_\xi(A_q + a_{q+1} L_\xi(\mu_{q+1})),$$
$$T_\xi(t_\xi(B_q), t_\xi(a_{q+1} G_\xi(\nu_{q+1}))) = t_\xi(B_q + a_{q+1} G_\xi(\nu_{q+1})).$$

Step 5: Complete the induction. The preceding expressions are

$$s_\xi\left(\sum_{j=1}^{q+1} a_j L_\xi(\mu_j)\right), \qquad t_\xi\left(\sum_{j=1}^{q+1} a_j G_\xi(\nu_j)\right),$$

so (13) holds for $q + 1$. Taking $a_j = \omega_j$ proves (12). Closure follows from repeated application of Theorem 3.1. □

Theorem 4.2 (Idempotency, boundedness, monotonicity, and permutation law for IFDADWA)**.** *For* $\sum_{j=1}^{r} \omega_j = 1$, *write* $\mathcal{A}_\xi(\boldsymbol{\gamma}, \boldsymbol{\omega}) = \mathrm{IFDADWA}_\xi(\boldsymbol{\gamma}; \boldsymbol{\omega})$. *Then*

$$\mathcal{A}_\xi((\gamma)_{j=1}^{r}, \boldsymbol{\omega}) = \gamma, \quad (14)$$
$$\gamma^- \preceq_{IF} \mathcal{A}_\xi(\boldsymbol{\gamma}, \boldsymbol{\omega}) \preceq_{IF} \gamma^+, \quad (15)$$
$$\boldsymbol{\gamma} \preceq_{IF} \boldsymbol{\gamma}' \Longrightarrow \mathcal{A}_\xi(\boldsymbol{\gamma}, \boldsymbol{\omega}) \preceq_{IF} \mathcal{A}_\xi(\boldsymbol{\gamma}', \boldsymbol{\omega}), \quad (16)$$
$$\mathcal{A}_\xi(\rho\boldsymbol{\gamma}, \rho\boldsymbol{\omega}) = \mathcal{A}_\xi(\boldsymbol{\gamma}, \boldsymbol{\omega}). \quad (17)$$

Here ρ *is any permutation of* $\{1, \ldots, r\}$, $\rho\boldsymbol{\gamma} = (\gamma_{\rho(1)}, \ldots, \gamma_{\rho(r)})$, *and* $\rho\boldsymbol{\omega} = (\omega_{\rho(1)}, \ldots, \omega_{\rho(r)})$.

4.2. *IF Dombi–Archimedean ordered weighted averaging operator*

Definition 4.2 (IFDADOWA ordering and closed form)**.** Sort the IFNs by Definition 2.2. Let σ satisfy $\gamma_{\sigma(1)} \succeq_* \cdots \succeq_* \gamma_{\sigma(r)}$. For position weights $w_j \geq 0$ with $\sum_j w_j = 1$, define

$$\mathrm{IFDADOWA}_\xi(\boldsymbol{\gamma}; \boldsymbol{w}) = \mathrm{IFDADWA}_\xi\Big((\gamma_{\sigma(j)})_{j=1}^{r}; \boldsymbol{w}\Big). \quad (18)$$

Its components are

$$\mu^{OWA} = s_\xi\left(\sum_{j=1}^{r} w_j L_\xi(\mu_{\sigma(j)})\right),$$
$$\nu^{OWA} = t_\xi\left(\sum_{j=1}^{r} w_j G_\xi(\nu_{\sigma(j)})\right), \quad (19)$$

so $\mathrm{IFDADOWA}_\xi = (\mu^{OWA}, \nu^{OWA})$.

Theorem 4.3 (Idempotency, boundedness, and matched-order monotonicity for IFDADOWA)**.**

$$\mathrm{IFDADOWA}_\xi(\gamma, \ldots, \gamma; \boldsymbol{w}) = \gamma, \quad (20)$$
$$\gamma^- \preceq_{IF} \mathrm{IFDADOWA}_\xi(\boldsymbol{\gamma}; \boldsymbol{w}) \preceq_{IF} \gamma^+, \quad (21)$$
$$\begin{bmatrix} \forall j: \\ \gamma_{\sigma(j)} \preceq_{IF} \gamma'_{\tau(j)} \end{bmatrix} \Longrightarrow \mathrm{IFDADOWA}_\xi(\boldsymbol{\gamma}; \boldsymbol{w}) \preceq_{IF} \mathrm{IFDADOWA}_\xi(\boldsymbol{\gamma}'; \boldsymbol{w}). \quad (22)$$

Moreover, $\mathrm{IFDADOWA}_\xi$ *is invariant under a permutation of the unsorted input list.*

4.3. *IF Dombi–Archimedean hybrid averaging operator*

Definition 4.3 (IFDADHA)**.** Let attribute weights $\Omega_j \geq 0$ and position weights $w_j \geq 0$ satisfy

$$\sum_{j=1}^{r} \Omega_j = 1, \qquad \sum_{j=1}^{r} w_j = 1,$$

and let $\eta \in [0, 1]$. Define

$$p_j = \eta\Omega_j + \frac{1-\eta}{r}, \qquad \widehat{\gamma}_j = (r p_j) \odot_{\mathcal{AD}} \gamma_j. \quad (23)$$

Sort $\widehat{\gamma}_j$ by the score–accuracy rule, obtaining σ, and define

$$h_j = \eta\Omega_{\sigma(j)} + (1-\eta) w_j, \qquad h_j \geq 0, \qquad \sum_{j=1}^{r} h_j = 1. \quad (24)$$

Then

$$\mathrm{IFDADHA}_\xi(\gamma_1, \ldots, \gamma_r; \boldsymbol{\Omega}, \boldsymbol{w}, \eta)$$
$$= \mathrm{IFDADWA}_\xi(\gamma_{\sigma(1)}, \ldots, \gamma_{\sigma(r)}; \boldsymbol{h})$$
$$= \left(s_\xi\left(\sum_{j=1}^{r} h_j L_\xi(\mu_{\sigma(j)})\right), t_\xi\left(\sum_{j=1}^{r} h_j G_\xi(\nu_{\sigma(j)})\right)\right). \quad (25)$$

At $\eta = 1$ it reduces to the attribute-weighted averaging operator; at $\eta = 0$ it reduces to the ordered weighted averaging operator.

Theorem 4.4 (Idempotency, boundedness, and qualified monotonicity for IFDADHA)**.**

$$\mathrm{IFDADHA}_\xi(\gamma,\dots,\gamma;\boldsymbol{\Omega},\boldsymbol{w},\eta)=\gamma, \tag{26}$$

$$\gamma^- \preceq_{IF} \mathrm{IFDADHA}_\xi(\boldsymbol{\gamma};\boldsymbol{\Omega},\boldsymbol{w},\eta)\preceq_{IF}\gamma^+, \tag{27}$$

$$\begin{bmatrix}\sigma=\sigma',\\ \boldsymbol{\gamma}\preceq_{IF}\boldsymbol{\gamma}'\end{bmatrix}\Longrightarrow \mathrm{IFDADHA}_\xi(\boldsymbol{\gamma};\boldsymbol{\Omega},\boldsymbol{w},\eta) \preceq_{IF} \mathrm{IFDADHA}_\xi(\boldsymbol{\gamma}';\boldsymbol{\Omega},\boldsymbol{w},\eta). \tag{28}$$

4.4. IF Dombi–Archimedean weighted geometric operator

Definition 4.4 (IFDADWG)**.** For normalized weights $\omega_j\ge 0$, $\sum_j\omega_j=1$, define

$$\mathrm{IFDADWG}_\xi(\gamma_1,\dots,\gamma_r;\boldsymbol{\omega})=\bigotimes_{j=1}^{r}\gamma_j^{\circledast_{\mathcal{AD}}\omega_j}. \tag{29}$$

Theorem 4.5 (Closed form of IFDADWG)**.** *Let*

$$\mu^{WG}=t_\xi\left(\sum_{j=1}^{r}\omega_jG_\xi(\mu_j)\right),$$

$$\nu^{WG}=s_\xi\left(\sum_{j=1}^{r}\omega_jL_\xi(\nu_j)\right). \tag{30}$$

Then

$$\mathrm{IFDADWG}_\xi(\boldsymbol{\gamma};\boldsymbol{\omega})=(\mu^{WG},\nu^{WG}). \tag{31}$$

Proof. Again prove the stronger claim for arbitrary $a_j\ge 0$:

$$\bigotimes_{j=1}^{r}\gamma_j^{\circledast_{\mathcal{AD}}a_j}=\left(t_\xi\left(\sum_{j=1}^{r}a_jG_\xi(\mu_j)\right),s_\xi\left(\sum_{j=1}^{r}a_jL_\xi(\nu_j)\right)\right). \tag{32}$$

Step 1: Base case. For $r=1$, Equation (9) gives (32).

Step 2: Induction hypothesis. Assume the claim for $r=q$ and put

$$C_q=\sum_{j=1}^{q}a_jG_\xi(\mu_j),\qquad D_q=\sum_{j=1}^{q}a_jL_\xi(\nu_j).$$

The product of the first q terms is $(t_\xi(C_q),s_\xi(D_q))$.

Step 3: Multiply by the next powered IFN.

$$\left[\bigotimes_{j=1}^{q}\gamma_j^{\circledast_{\mathcal{AD}}a_j}\right]\otimes_{\mathcal{AD}}\gamma_{q+1}^{\circledast_{\mathcal{AD}}a_{q+1}}$$
$$=\Big(T_\xi(t_\xi(C_q),t_\xi(a_{q+1}G_\xi(\mu_{q+1}))),S_\xi(s_\xi(D_q),s_\xi(a_{q+1}L_\xi(\nu_{q+1})))\Big).$$

Step 4: Use $G_\xi(t_\xi(z))=z$ and $L_\xi(s_\xi(z))=z$.

$$T_\xi(t_\xi(C_q),t_\xi(a_{q+1}G_\xi(\mu_{q+1})))=t_\xi(C_q+a_{q+1}G_\xi(\mu_{q+1})),$$
$$S_\xi(s_\xi(D_q),s_\xi(a_{q+1}L_\xi(\nu_{q+1})))=s_\xi(D_q+a_{q+1}L_\xi(\nu_{q+1})).$$

Step 5: Complete the induction. These are the sums through $q+1$, proving (32). Setting $a_j=\omega_j$ yields (30); closure follows from Theorem 3.1. □

Example 4.1 (Complete geometric calculation)**.** Let $\gamma_1=(0.65,0.20)$, $\gamma_2=(0.50,0.30)$, and $\gamma_3=(0.35,0.45)$. With $\boldsymbol{\omega}=(0.30,0.40,0.30)$ and $\xi=2$, Equation (30) gives

$$\mathrm{IFDADWG}_2(\gamma_1,\gamma_2,\gamma_3;\boldsymbol{\omega})=(0.4477,0.3512),$$
$$0.4477+0.3512=0.7989\le 1.$$

Theorem 4.6 (Idempotency, boundedness, monotonicity, and permutation law for IFDADWG)**.** *Write* $\mathcal{G}_\xi(\boldsymbol{\gamma},\boldsymbol{\omega})=\mathrm{IFDADWG}_\xi(\boldsymbol{\gamma};\boldsymbol{\omega})$. *Then*

$$\mathcal{G}_\xi((\gamma)_{j=1}^r,\boldsymbol{\omega})=\gamma, \tag{33}$$

$$\gamma^-\preceq_{IF}\mathcal{G}_\xi(\boldsymbol{\gamma},\boldsymbol{\omega})\preceq_{IF}\gamma^+, \tag{34}$$

$$\boldsymbol{\gamma}\preceq_{IF}\boldsymbol{\gamma}'\Longrightarrow\mathcal{G}_\xi(\boldsymbol{\gamma},\boldsymbol{\omega})\preceq_{IF}\mathcal{G}_\xi(\boldsymbol{\gamma}',\boldsymbol{\omega}), \tag{35}$$

$$\mathcal{G}_\xi(\rho\boldsymbol{\gamma},\rho\boldsymbol{\omega})=\mathcal{G}_\xi(\boldsymbol{\gamma},\boldsymbol{\omega}). \tag{36}$$

4.5. IF Dombi–Archimedean ordered weighted geometric operator

Definition 4.5 (IFDADOWG)**.** Let σ be the score–accuracy sorting permutation and let $w_j\ge 0$, $\sum_j w_j=1$. Define

$$\mathrm{IFDADOWG}_\xi(\boldsymbol{\gamma};\boldsymbol{w})=\mathrm{IFDADWG}_\xi\Big((\gamma_{\sigma(j)})_{j=1}^r;\boldsymbol{w}\Big). \tag{37}$$

Its components are

$$\mu^{OWG}=t_\xi\left(\sum_{j=1}^{r}w_jG_\xi(\mu_{\sigma(j)})\right),$$

$$\nu^{OWG}=s_\xi\left(\sum_{j=1}^{r}w_jL_\xi(\nu_{\sigma(j)})\right), \tag{38}$$

so $\mathrm{IFDADOWG}_\xi=(\mu^{OWG},\nu^{OWG})$.

Theorem 4.7 (Idempotency, boundedness, and matched-order monotonicity for IFDADOWG)**.**

$$\mathrm{IFDADOWG}_\xi(\gamma,\dots,\gamma;\boldsymbol{w})=\gamma, \tag{39}$$

$$\gamma^-\preceq_{IF}\mathrm{IFDADOWG}_\xi(\boldsymbol{\gamma};\boldsymbol{w})\preceq_{IF}\gamma^+, \tag{40}$$

$$\begin{bmatrix}\forall j:\\ \gamma_{\sigma(j)}\preceq_{IF}\gamma'_{\tau(j)}\end{bmatrix}\Longrightarrow\mathrm{IFDADOWG}_\xi(\boldsymbol{\gamma};\boldsymbol{w}) \preceq_{IF}\mathrm{IFDADOWG}_\xi(\boldsymbol{\gamma}';\boldsymbol{w}). \tag{41}$$

Moreover, $\mathrm{IFDADOWG}_\xi$ *is invariant under a permutation of the unsorted input list.*

4.6. IF Dombi–Archimedean hybrid geometric operator

Definition 4.6 (IFDADHG)**.** Let $\boldsymbol{\Omega}$, $\boldsymbol{w}$, and η satisfy the normalization conditions in Definition 4.3. Set

$$p_j=\eta\Omega_j+\frac{1-\eta}{r},\qquad \widehat{\gamma}_j=\gamma_j^{\circledast_{\mathcal{AD}}(rp_j)}. \tag{42}$$

Sort $\widehat{\gamma}_j$, obtaining σ, and set

$$h_j=\eta\Omega_{\sigma(j)}+(1-\eta)w_j,\qquad \sum_{j=1}^{r}h_j=1.$$

Then

$$\begin{aligned}&\mathrm{IFDADHG}_{\xi}(\gamma_1,\ldots,\gamma_r;\boldsymbol{\Omega},\boldsymbol{w},\eta)\\&=\mathrm{IFDADWG}_{\xi}(\gamma_{\sigma(1)},\ldots,\gamma_{\sigma(r)};\boldsymbol{h})\\&=\left(t_{\xi}\left(\sum_{j=1}^{r}h_jG_{\xi}(\mu_{\sigma(j)})\right),s_{\xi}\left(\sum_{j=1}^{r}h_jL_{\xi}(\nu_{\sigma(j)})\right)\right).\end{aligned}\tag{43}$$

At $\eta = 1$ it reduces to the attribute-weighted geometric operator; at $\eta = 0$ it reduces to the ordered weighted geometric operator.

Theorem 4.8 (Idempotency, boundedness, and qualified monotonicity for IFDADHG)**.**

$$\mathrm{IFDADHG}_{\xi}(\gamma,\ldots,\gamma;\boldsymbol{\Omega},\boldsymbol{w},\eta)=\gamma,\tag{44}$$

$$\gamma^{-}\preceq_{IF}\mathrm{IFDADHG}_{\xi}(\boldsymbol{\gamma};\boldsymbol{\Omega},\boldsymbol{w},\eta)\preceq_{IF}\gamma^{+},\tag{45}$$

$$\begin{bmatrix}\sigma=\sigma',\\ \boldsymbol{\gamma}\preceq_{IF}\boldsymbol{\gamma}'\end{bmatrix}\Longrightarrow\mathrm{IFDADHG}_{\xi}(\boldsymbol{\gamma};\boldsymbol{\Omega},\boldsymbol{w},\eta)\preceq_{IF}\mathrm{IFDADHG}_{\xi}(\boldsymbol{\gamma}';\boldsymbol{\Omega},\boldsymbol{w},\eta).\tag{46}$$

Proposition 4.1 (Closure of the ordered and hybrid constructions)**.** *Each output in Equations* (19)*,* (25)*,* (38)*, and* (43) *is an IFN.*

Proof. **Step 1.** The sorting rule only permutes valid IFNs and therefore cannot change their feasibility.

Step 2. The transformed objects in (23) and (42) are IFNs by Theorem 3.1, because $rp_j \geq 0$.

Step 3. From $\sum_j \Omega_j = \sum_j w_j = 1$,

$$\sum_{j=1}^{r}h_j=\eta\sum_{j=1}^{r}\Omega_{\sigma(j)}+(1-\eta)\sum_{j=1}^{r}w_j=\eta+(1-\eta)=1.$$

Thus every ordered or hybrid output is an IFDADWA or IFDADWG result with a normalized nonnegative weight vector.

Step 4. Theorems 4.1 and 4.5, together with Theorem 3.1, imply that all four outputs satisfy $0 \leq \mu + \nu \leq 1$. □

Remark 4.1. The complete operator family is

$$\{\mathrm{IFDADWA},\mathrm{IFDADOWA},\mathrm{IFDADHA},\\ \mathrm{IFDADWG},\mathrm{IFDADOWG},\mathrm{IFDADHG}\}.$$

The idempotency, boundedness, and monotonicity results above are intentionally presented as mathematical theorem statements without appended proofs, while closure and the two closed-form derivations are proved step by step.

5. Proposed IFDA–RANCOM–AROMAN decision framework

Consider alternatives $\overline{A}_i$, $i = 1,\ldots,m$, criteria $\overline{C}_j$, $j = 1,\ldots,n$, and experts E_e, $e = 1,\ldots,p$. Let J_B and J_C denote the benefit- and cost-criterion index sets, respectively, with $J_B \cap J_C = \varnothing$ and $J_B \cup J_C = \{1,\ldots,n\}$. Expert weights satisfy

$$v_e \geq 0,\qquad \sum_{e=1}^{p}v_e=1.\tag{47}$$

The single Dombi flexibility parameter is $\xi > 0$, the normalization-fusion coefficient is $\beta \in [0,1]$, and the final AROMAN coefficient is $\lambda \in [0,1]$.

5.1. Computational formulas

For $\gamma_j = (\mu_j,\nu_j)$ and a normalized vector $\boldsymbol{\omega}$, define

$$M_{\mu}=\sum_{j=1}^{r}\omega_j\left(\frac{\mu_j}{1-\mu_j}\right)^{\xi},\qquad N_{\nu}=\sum_{j=1}^{r}\omega_j\left(\frac{1-\nu_j}{\nu_j}\right)^{\xi},$$

$$N_{\mu}=\sum_{j=1}^{r}\omega_j\left(\frac{1-\mu_j}{\mu_j}\right)^{\xi},\qquad M_{\nu}=\sum_{j=1}^{r}\omega_j\left(\frac{\nu_j}{1-\nu_j}\right)^{\xi}.$$

The implementable averaging and geometric equations are

$$\mathrm{IFDADWA}_{\xi}(\gamma_1,\ldots,\gamma_r;\boldsymbol{\omega})=\left(\frac{M_{\mu}^{1/\xi}}{1+M_{\mu}^{1/\xi}},\frac{1}{1+N_{\nu}^{1/\xi}}\right),\tag{48}$$

$$\mathrm{IFDADWG}_{\xi}(\gamma_1,\ldots,\gamma_r;\boldsymbol{\omega})=\left(\frac{1}{1+N_{\mu}^{1/\xi}},\frac{M_{\nu}^{1/\xi}}{1+M_{\nu}^{1/\xi}}\right).\tag{49}$$

The endpoint conventions in Definition 2.5 are used whenever a component is 0 or 1. Equations (48) and (49) are the exact formulas used in all numerical tables.

5.2. Proposed IFDA–RANCOM criterion-weighting method

The RANCOM component is carried out independently before the AROMAN ranking stage.

Step R1: Construct the criterion-importance IF information.. Expert E_e evaluates the importance of criterion $\overline{C}_j$ by

$$\widetilde{g}_j^{(e)}=\left(\mu_j^{(e)},\nu_j^{(e)}\right),\qquad 0\leq\mu_j^{(e)}+\nu_j^{(e)}\leq 1.\tag{50}$$

Step R2: Aggregate the expert criterion judgments.. For each j, compute

$$\widetilde{g}_j=\mathrm{IFDADWA}_{\xi}\left(\widetilde{g}_j^{(1)},\ldots,\widetilde{g}_j^{(p)};\boldsymbol{v}\right)=(\mu_j^g,\nu_j^g).\tag{51}$$

The full computational form is

$$M_j^g=\sum_{e=1}^{p}v_e\left(\frac{\mu_j^{(e)}}{1-\mu_j^{(e)}}\right)^{\xi},\qquad N_j^g=\sum_{e=1}^{p}v_e\left(\frac{1-\nu_j^{(e)}}{\nu_j^{(e)}}\right)^{\xi},\tag{52}$$

$$\mu_j^g=\frac{(M_j^g)^{1/\xi}}{1+(M_j^g)^{1/\xi}},\qquad \nu_j^g=\frac{1}{1+(N_j^g)^{1/\xi}}.\tag{53}$$

Proposition 5.1 (IF validity of aggregated criterion importance)**.** *For every criterion $\overline{C}_j$, the pair $\widetilde{g}_j$ in* (51) *satisfies* $0 \leq \mu_j^g + \nu_j^g \leq 1$.

Proof. **Step 1.** From (50), $\mu_j^{(e)} \leq 1-\nu_j^{(e)}$ for every expert e.

Step 2. Since L_{ξ} is increasing,

$$L_{\xi}(\mu_j^{(e)})\leq L_{\xi}(1-\nu_j^{(e)})=G_{\xi}(\nu_j^{(e)}).$$

Step 3. Multiply by $v_e \geq 0$ and sum over all experts:

$$M_j^g=\sum_{e}v_eL_{\xi}(\mu_j^{(e)})\leq\sum_{e}v_eG_{\xi}(\nu_j^{(e)})=N_j^g.$$

Step 4. Put $a = (M_j^g)^{1/\xi}$ and $b = (N_j^g)^{1/\xi}$. Then $0 \le a \le b$, and

$$\mu_j^g + \nu_j^g = \frac{a}{1+a} + \frac{1}{1+b} \le \frac{a}{1+a} + \frac{1}{1+a} = 1.$$

Both components belong to $[0,1]$, so $\widetilde{g}_j$ is an IFN. □

Step R3: Obtain a bounded importance index and criterion order.. Calculate

$$q_j = \frac{1 + \mathscr{S}(\widetilde{g}_j)}{2} = \frac{1 + \mu_j^g - \nu_j^g}{2} \in [0,1], \qquad h_j^g = \mathscr{H}(\widetilde{g}_j) = \mu_j^g + \nu_j^g. \tag{54}$$

For two criteria, define

$$\overline{C}_j \succ_R \overline{C}_k \Longleftrightarrow \left[q_j > q_k\right] \text{ or } \left[q_j = q_k \text{ and } h_j^g > h_k^g\right]. \tag{55}$$

If both q and h^g are equal, the criteria are tied; the original criterion index is used only for deterministic display.

Step R4: Construct the RANCOM relation matrix.. Using the order in (55), define

$$r_{jk} = \begin{cases} 1, & \overline{C}_j \succ_R \overline{C}_k, \\ \frac{1}{2}, & \overline{C}_j \sim_R \overline{C}_k, \\ 0, & \overline{C}_j \prec_R \overline{C}_k. \end{cases} \tag{56}$$

The diagonal entries satisfy $r_{jj} = 1/2$.

Step R5: Calculate and normalize the RANCOM weights.. Let

$$s_j = \sum_{k=1}^{n} r_{jk}, \qquad \omega_j = \frac{s_j}{\sum_{\ell=1}^{n} s_\ell}. \tag{57}$$

Proposition 5.2 (Validity of the IFDA–RANCOM weight vector)**.** *The weights in* (57) *satisfy*

$$\omega_j \ge 0, \qquad \sum_{j=1}^{n} \omega_j = 1. \tag{58}$$

Proof. **Step 1.** Equation (56) gives $r_{jk} \in \{0, 1/2, 1\}$, hence $s_j = \sum_k r_{jk} \ge 0$.

Step 2. Since $r_{jj} = 1/2$,

$$\sum_{\ell=1}^{n} s_\ell = \sum_{\ell=1}^{n}\sum_{k=1}^{n} r_{\ell k} \ge \sum_{\ell=1}^{n} r_{\ell\ell} = \frac{n}{2} > 0.$$

Thus the denominator in (57) is strictly positive.

Step 3. Therefore $\omega_j = s_j/(\sum_\ell s_\ell) \ge 0$.

Step 4. Finally,

$$\sum_{j=1}^{n} \omega_j = \frac{\sum_{j=1}^{n} s_j}{\sum_{\ell=1}^{n} s_\ell} = 1.$$

□

5.3. Proposed IFDA–AROMAN alternative-ranking method

After the RANCOM weights have been obtained, the AROMAN component ranks the alternatives through the following independent stages.

Step A1: Form the expert decision matrices.. Let $x_{ij}^{(e)} \ge 0$ be the evaluation of alternative $\overline{A}_i$ under criterion $\overline{C}_j$ by expert E_e. For each criterion and expert, define

$$x_{j,-}^{(e)} = \min_i x_{ij}^{(e)}, \qquad x_{j,+}^{(e)} = \max_i x_{ij}^{(e)}.$$

Step A2: Calculate the two AROMAN normalizations separately.. The linear normalization is

$$\ell_{ij}^{(e)} = \begin{cases} \dfrac{x_{ij}^{(e)} - x_{j,-}^{(e)}}{x_{j,+}^{(e)} - x_{j,-}^{(e)}}, & x_{j,+}^{(e)} > x_{j,-}^{(e)}, \\ 1, & x_{j,+}^{(e)} = x_{j,-}^{(e)}, \end{cases} \tag{59}$$

and the vector normalization is

$$v_{ij}^{(e)} = \begin{cases} \dfrac{x_{ij}^{(e)}}{\sqrt{\sum_{r=1}^{m} (x_{rj}^{(e)})^2}}, & \sum_{r=1}^{m} (x_{rj}^{(e)})^2 > 0, \\ 0, & \sum_{r=1}^{m} (x_{rj}^{(e)})^2 = 0. \end{cases} \tag{60}$$

Thus $\ell_{ij}^{(e)}, v_{ij}^{(e)} \in [0,1]$. Cost criteria are not reversed at this stage.

Step A3: Map each normalized value to an IFN.. For $x \in [0,1]$, define

$$p(x) = \varepsilon + (1 - 2\varepsilon)x, \qquad \pi(x) = \pi_0 + 4(\pi_{\max} - \pi_0)x(1-x), \tag{61}$$

where

$$0 < \varepsilon < \frac{1}{2}, \qquad 0 \le \pi_0 \le \pi_{\max} < 1.$$

The IF embedding is

$$\Phi(x) = \left([1 - \pi(x)]p(x), [1 - \pi(x)][1 - p(x)]\right). \tag{62}$$

Set

$$\widetilde{\ell}_{ij}^{(e)} = \Phi(\ell_{ij}^{(e)}), \qquad \widetilde{v}_{ij}^{(e)} = \Phi(v_{ij}^{(e)}). \tag{63}$$

Proposition 5.3 (Validity of the IF embedding)**.** *For every $x \in [0,1]$, $\Phi(x)$ is an IFN.*

Proof. **Step 1.** Since $0 \le x \le 1$ and $0 < \varepsilon < 1/2$,

$$\varepsilon \le p(x) \le 1 - \varepsilon,$$

so $p(x), 1 - p(x) \in [0,1]$.

Step 2. Since $0 \le x(1-x) \le 1/4$,

$$\pi_0 \le \pi(x) \le \pi_{\max} < 1.$$

Thus $1 - \pi(x) \in (0,1]$.

Step 3. Both components of $\Phi(x)$ are nonnegative and

$$[1 - \pi(x)]p(x) + [1 - \pi(x)][1 - p(x)] = 1 - \pi(x) \le 1.$$

Therefore $\Phi(x)$ is an IFN with hesitation $\pi(x)$. □

Step A4: Aggregate the experts for each normalization.. Apply the IFDADWA operator separately:

$$\widetilde{\ell}_{ij} = \mathrm{IFDADWA}_{\xi}\Big(\widetilde{\ell}^{(1)}_{ij}, \ldots, \widetilde{\ell}^{(p)}_{ij}; \boldsymbol{\nu}\Big), \quad \widetilde{v}_{ij} = \mathrm{IFDADWA}_{\xi}\Big(\widetilde{v}^{(1)}_{ij}, \ldots, \widetilde{v}^{(p)}_{ij}; \boldsymbol{\nu}\Big). \tag{64}$$

Theorem 4.1 and Proposition 5.3 imply that both aggregated matrices consist entirely of IFNs.

Step A5: Fuse the two normalized IF matrices.. For $\beta \in [0, 1]$, define

$$\widetilde{n}^{DA}_{ij} = \mathrm{IFDADWA}_{\xi}\left(\widetilde{\ell}_{ij}, \widetilde{v}_{ij}; \beta, 1-\beta\right) = (\mu^N_{ij}, \nu^N_{ij}). \tag{65}$$

Writing $\widetilde{\ell}_{ij} = (\mu_L, \nu_L)$ and $\widetilde{v}_{ij} = (\mu_V, \nu_V)$ gives

$$\mu^N_{ij} = 1 - \frac{1}{1 + \left[\beta\left(\frac{\mu_L}{1-\mu_L}\right)^{\xi} + (1-\beta)\left(\frac{\mu_V}{1-\mu_V}\right)^{\xi}\right]^{1/\xi}}, \tag{66}$$

$$\nu^N_{ij} = \frac{1}{1 + \left[\beta\left(\frac{1-\nu_L}{\nu_L}\right)^{\xi} + (1-\beta)\left(\frac{1-\nu_V}{\nu_V}\right)^{\xi}\right]^{1/\xi}}. \tag{67}$$

Proposition 5.4 (Closure and boundary reduction of the normalization fusion)**.**

$$\widetilde{n}^{DA}_{ij} \in IF, \tag{68}$$

$$\beta = 1 \Longrightarrow \widetilde{n}^{DA}_{ij} = \widetilde{\ell}_{ij}, \tag{69}$$

$$\beta = 0 \Longrightarrow \widetilde{n}^{DA}_{ij} = \widetilde{v}_{ij}. \tag{70}$$

Proof. **Step 1.** Since $\widetilde{\ell}_{ij}$ and $\widetilde{v}_{ij}$ are IFNs,

$$L_{\xi}(\mu_L) \le G_{\xi}(\nu_L), \qquad L_{\xi}(\mu_V) \le G_{\xi}(\nu_V).$$

Step 2. Multiply the first pair by $\beta \ge 0$, the second by $1-\beta \ge 0$, and add:

$$\beta L_{\xi}(\mu_L) + (1-\beta)L_{\xi}(\mu_V) \le \beta G_{\xi}(\nu_L) + (1-\beta)G_{\xi}(\nu_V).$$

Step 3. Let the left and right sides be A and B, respectively. Then $A \le B$, so

$$s_{\xi}(A) + t_{\xi}(B) \le s_{\xi}(A) + t_{\xi}(A) = 1.$$

Equations (66)–(67) are precisely $(s_{\xi}(A), t_{\xi}(B))$; hence the fused pair is an IFN.

Step 4. If $\beta = 1$, then $A = L_{\xi}(\mu_L)$ and $B = G_{\xi}(\nu_L)$, so the inverse identities give (μ_L, ν_L). If $\beta = 0$, the same calculation gives (μ_V, ν_V). □

Step A6: Convert cost criteria and calculate IF utilities.. The IF complement is

$$(\mu, \nu)^c = (\nu, \mu). \tag{71}$$

Define

$$W_B = \sum_{j \in J_B} \omega_j, \qquad W_C = \sum_{j \in J_C} \omega_j, \qquad W_B + W_C = 1. \tag{72}$$

When both sets are nonempty, use

$$\widehat{\omega}^B_j = \frac{\omega_j}{W_B}, \quad j \in J_B, \qquad \widehat{\omega}^C_j = \frac{\omega_j}{W_C}, \quad j \in J_C, \tag{73}$$

so that each within-group vector sums to one. The benefit and cost-desirability IFNs are

$$\widetilde{B}_i = \mathrm{IFDADWA}_{\xi}\left(\widetilde{n}^{DA}_{ij} : j \in J_B; \widehat{\omega}^B_j : j \in J_B\right), \tag{74}$$

$$\widetilde{C}_i = \mathrm{IFDADWA}_{\xi}\left((\widetilde{n}^{DA}_{ij})^c : j \in J_C; \widehat{\omega}^C_j : j \in J_C\right). \tag{75}$$

Proposition 5.5 (Closure of benefit and cost-desirability utilities)**.** *For every alternative $\widetilde{A}_i$, both $\widetilde{B}_i$ and $\widetilde{C}_i$ are IFNs.*

Proof. **Step 1.** Proposition 5.4 gives $\widetilde{n}^{DA}_{ij} \in IF$.

Step 2. If (μ, ν) is an IFN, then (ν, μ) is also an IFN because $\nu + \mu = \mu + \nu \le 1$. Hence every cost-desirability input in (75) is valid.

Step 3. Equation (73) gives $\sum_{j \in J_B} \widehat{\omega}^B_j = 1$ and $\sum_{j \in J_C} \widehat{\omega}^C_j = 1$.

Step 4. Theorem 4.1 therefore applies to both groups, proving that $\widetilde{B}_i$ and $\widetilde{C}_i$ satisfy the IF feasibility condition. □

Step A7: Convert the IF utilities to bounded scalar components.. Set

$$b_i = \frac{1 + \mathscr{S}(\widetilde{B}_i)}{2}, \qquad c_i = \frac{1 + \mathscr{S}(\widetilde{C}_i)}{2}, \tag{76}$$

and retain the criterion-type masses through

$$A_i = W_B b_i, \qquad L_i = W_C c_i. \tag{77}$$

Step A8: Calculate the modified AROMAN appraisal.. The final appraisal is

$$R_i = L_i^{\lambda} + A_i^{1-\lambda}, \qquad 0 \le \lambda \le 1. \tag{78}$$

A larger R_i indicates a more desirable alternative. An interpretable baseline is

$$\lambda = \frac{|J_C|}{n}, \tag{79}$$

but it remains a modeling choice and must be examined by sensitivity analysis.

Theorem 5.1 (Boundedness and monotonicity of the IFDA utility stage)**.**

$$0 \le b_i, c_i, A_i, L_i \le 1, \tag{80}$$

$$\left[\forall j \in J_B : \widetilde{n}^{DA}_{ij} \le_{IF} \widetilde{n}'^{DA}_{ij}, \quad \forall j \in J_C : (\widetilde{n}^{DA}_{ij})^c \le_{IF} (\widetilde{n}'^{DA}_{ij})^c\right] \Longrightarrow A_i \le A'_i, \qquad L_i \le L'_i. \tag{81}$$

Theorem 5.2 (Monotonicity of the final AROMAN appraisal)**.** *For $0 < \lambda < 1$ and $A_i, L_i > 0$,*

$$\frac{\partial R_i}{\partial A_i} = (1-\lambda)A_i^{-\lambda} > 0, \qquad \frac{\partial R_i}{\partial L_i} = \lambda L_i^{\lambda-1} > 0. \tag{82}$$

Consequently,

$$A_i \le A'_i, \ L_i \le L'_i \Longrightarrow R_i \le R'_i. \tag{83}$$

The endpoint cases $\lambda \in \{0, 1\}$ are nondecreasing in the component retained by (78).

Algorithm 1 Proposed IFDA–RANCOM criterion-weighting procedure

Require: Criterion-importance IFNs $\widetilde{g}_j^{(e)}$, expert weights $\boldsymbol{v}$, and $\xi > 0$.
Ensure: Normalized criterion-weight vector $\boldsymbol{\omega}$.

for $j = 1, \ldots, n$ **do**
  Calculate M_j^g and N_j^g from (52).
  Calculate $\widetilde{g}_j = (\mu_j^g, \nu_j^g)$ from (53).
  Calculate q_j and h_j^g from (54).
end for
Order the criteria by (55).
Construct $R = [r_{jk}]$ using (56).
Calculate s_j and ω_j using (57).
Verify $\omega_j \geq 0$ and $\sum_j \omega_j = 1$.

Algorithm 2 Proposed IFDA–AROMAN alternative-ranking procedure

Require: Decision matrices $X^{(e)}$, expert weights $\boldsymbol{v}$, RANCOM weights $\boldsymbol{\omega}$, criterion types, ξ, β, and λ.
Ensure: Appraisal values R_i and final ranking.

Calculate $\ell_{ij}^{(e)}$ and $\nu_{ij}^{(e)}$ from (59)–(60).
Map both normalized matrices to IFNs using (61)–(62).
Aggregate experts separately using (64).
Fuse the two IF matrices using (66)–(67).
Complement each cost-criterion IFN using (71).
Calculate W_B, W_C and the within-group weights from (72)–(73).
Calculate $\widetilde{B}_i$ and $\widetilde{C}_i$ from (74)–(75).
Calculate b_i, c_i, A_i, L_i from (76)–(77).
Calculate R_i from (78).
Rank the alternatives in descending order of R_i.
Retain the sensitivity and comparative analyses in Section 7.

5.4. Separate computational algorithms

6. Hypothetical case study: AI-enabled telemedicine platform selection

6.1. Alternatives, criteria, and raw data

Six illustrative platform profiles are considered: $\overline{A}_1$ cloud diagnostic support, $\overline{A}_2$ AI triage with EHR integration, $\overline{A}_3$ mobile-first rural telemedicine, $\overline{A}_4$ low-cost video consultation, $\overline{A}_5$ privacy-preserving federated AI, and $\overline{A}_6$ community-health-worker assisted telemedicine. The criteria are reported in Table 2.

The numerical values are hypothetical and demonstrate the complete computation; they are not clinical-performance claims. Three experts are assigned weights

$$\boldsymbol{v} = (0.40, 0.35, 0.25).$$

For reproducibility, their normalized assessments are represented by

$$\begin{aligned} x^{(1)} &= x, \\ x^{(2)} &= \min\{1, \max\{0, 0.97x + 0.015\}\}, \\ x^{(3)} &= \min\{1, \max\{0, 1.03x - 0.015\}\}. \end{aligned} \tag{84}$$

The IF mapping uses $\varepsilon = 0.10$, $\pi_0 = 0.05$, and $\pi_{\max} = 0.15$.

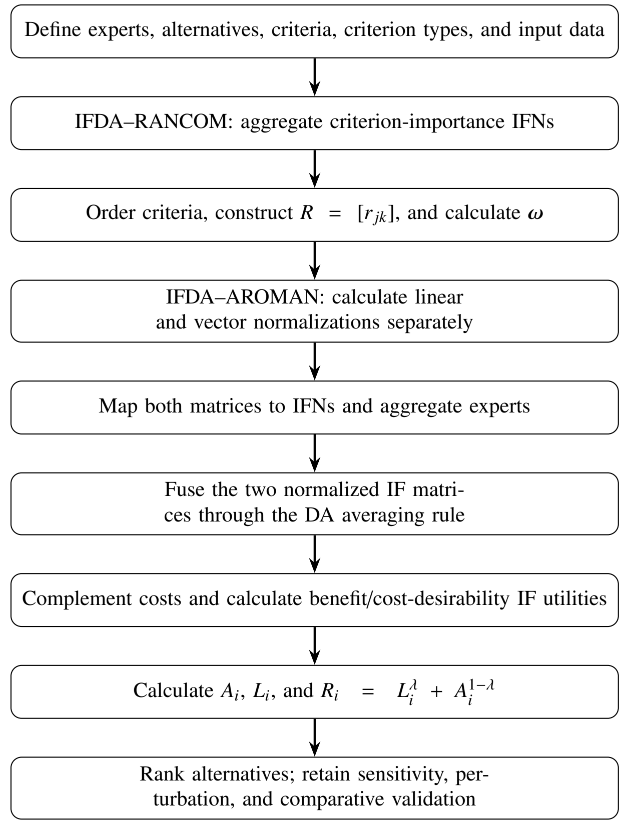

Figure 1. Separated IFDA–RANCOM weighting and IFDA–AROMAN ranking workflow.

6.2. IFDA–RANCOM criterion weights

The expert importance judgments, aggregated IFNs, normalized scores, and final RANCOM weights are given in Table 4. The group scores yield

$$\overline{C}_1 \succ \overline{C}_2 \succ \overline{C}_3 \succ \overline{C}_4 \succ \overline{C}_5.$$

The corresponding strict RANCOM row sums are

$$(4.5, 3.5, 2.5, 1.5, 0.5),$$

giving

$$\boldsymbol{\omega} = (0.36, 0.28, 0.20, 0.12, 0.04). \tag{85}$$

6.3. Dombi–Archimedean fusion of the two normalized IF matrices

At the illustrative baseline $\xi = 2$ and $\beta = 0.5$, Table 5 reports the proposed Dombi–Archimedean fusion of the linear and vector normalized IF matrices. Cost entries are still shown in their original magnitude orientation; they are complemented only when (75) is applied.

Table 2. Telemedicine selection criteria.

Criterion	Description	Unit	Type
$\overline{C}_1$	Diagnostic decision-support accuracy	percentage	Benefit
$\overline{C}_2$	Privacy/security compliance	score out of 100	Benefit
$\overline{C}_3$	EHR interoperability	score out of 100	Benefit
$\overline{C}_4$	Implementation cost	thousand monetary units	Cost
$\overline{C}_5$	Training/deployment burden	staff-hours	Cost

Table 3. Illustrative raw decision matrix.

Alternative	$\overline{C}_1$	$\overline{C}_2$	$\overline{C}_3$	$\overline{C}_4$	$\overline{C}_5$
$\overline{A}_1$	86	82	78	420	32
$\overline{A}_2$	91	88	84	510	40
$\overline{A}_3$	83	79	91	360	28
$\overline{A}_4$	78	72	75	290	22
$\overline{A}_5$	89	94	87	560	45
$\overline{A}_6$	80	76	69	240	18

Table 4. IFDA aggregation of criterion importance and RANCOM weights.

Criterion	E_1	E_2	E_3	Group IFN	q_j	ω_j
$\overline{C}_1$	$(0.90, 0.05)$	$(0.88, 0.07)$	$(0.85, 0.10)$	$(0.8850, 0.0623)$	0.9114	0.3600
$\overline{C}_2$	$(0.82, 0.10)$	$(0.80, 0.12)$	$(0.78, 0.15)$	$(0.8050, 0.1150)$	0.8450	0.2800
$\overline{C}_3$	$(0.75, 0.15)$	$(0.72, 0.18)$	$(0.70, 0.20)$	$(0.7295, 0.1692)$	0.7802	0.2000
$\overline{C}_4$	$(0.65, 0.22)$	$(0.62, 0.25)$	$(0.60, 0.28)$	$(0.6290, 0.2418)$	0.6936	0.1200
$\overline{C}_5$	$(0.55, 0.32)$	$(0.52, 0.35)$	$(0.50, 0.38)$	$(0.5287, 0.3427)$	0.5930	0.0400

Table 5. Dombi–Archimedean fused IF decision matrix.

Alternative	$\overline{C}_1$	$\overline{C}_2$	$\overline{C}_3$	$\overline{C}_4$	$\overline{C}_5$
$\overline{A}_1$	$(0.455, 0.397)$	$(0.380, 0.472)$	$(0.360, 0.494)$	$(0.428, 0.423)$	$(0.405, 0.445)$
$\overline{A}_2$	$(0.801, 0.133)$	$(0.528, 0.332)$	$(0.496, 0.360)$	$(0.629, 0.253)$	$(0.603, 0.272)$
$\overline{A}_3$	$(0.354, 0.500)$	$(0.332, 0.526)$	$(0.801, 0.133)$	$(0.337, 0.520)$	$(0.335, 0.523)$
$\overline{A}_4$	$(0.274, 0.594)$	$(0.264, 0.607)$	$(0.315, 0.546)$	$(0.252, 0.626)$	$(0.246, 0.633)$
$\overline{A}_5$	$(0.627, 0.255)$	$(0.801, 0.133)$	$(0.602, 0.274)$	$(0.802, 0.132)$	$(0.802, 0.132)$
$\overline{A}_6$	$(0.296, 0.569)$	$(0.296, 0.568)$	$(0.259, 0.614)$	$(0.203, 0.691)$	$(0.198, 0.698)$

6.4. Benefit/cost utilities and final ranking

Here $W_B = 0.84$, $W_C = 0.16$, and the baseline AROMAN coefficient is

$$\lambda = \frac{|J_C|}{5} = \frac{2}{5} = 0.4.$$

Table 6 reports every final component.

The final order is

$$\boxed{\overline{A}_3 \succ \overline{A}_2 \succ \overline{A}_5 \succ \overline{A}_1 \succ \overline{A}_6 \succ \overline{A}_4}. \tag{86}$$

Alternative $\overline{A}_3$ is preferred because its strongest interoperability performance is combined with moderate cost and training burden. Alternatives $\overline{A}_2$ and $\overline{A}_5$ possess stronger diagnostic/privacy profiles but receive lower cost-desirability utilities. This interpretation is a direct consequence of separating the benefit and cost IFDA components rather than collapsing all criteria into one score at the beginning.

7. Sensitivity and comparative analysis

7.1. Sensitivity to the Dombi parameter

Table 7 varies ξ. The most flexible setting $\xi = 0.5$ interchanges the first two alternatives, while $\overline{A}_3$ is first for every tested $\xi \geq 1$. The top-three set remains $\{\overline{A}_2, \overline{A}_3, \overline{A}_5\}$ throughout, demonstrating stability of the shortlist but also showing that the Dombi parameter is a substantive decision attitude rather than a decorative constant.

Table 6. IFDA utilities and modified AROMAN appraisal values.

Alt.	$\widetilde{B}_i$	$\widetilde{C}_i$	b_i	c_i	A_i	L_i	R_i	Rank
$\overline{A}_1$	$(0.4139, 0.4373)$	$(0.4289, 0.4217)$	0.4883	0.5036	0.4102	0.0806	0.9510	4
$\overline{A}_2$	$(0.7337, 0.1813)$	$(0.2582, 0.6220)$	0.7762	0.3181	0.6520	0.0509	1.0775	2
$\overline{A}_3$	$(0.6687, 0.2327)$	$(0.5209, 0.3363)$	0.7180	0.5923	0.6031	0.0948	1.1280	1
$\overline{A}_4$	$(0.2824, 0.5844)$	$(0.6281, 0.2502)$	0.3490	0.6890	0.2932	0.1102	0.8928	6
$\overline{A}_5$	$(0.7281, 0.1843)$	$(0.1320, 0.8019)$	0.7719	0.1650	0.6484	0.0264	1.0048	3
$\overline{A}_6$	$(0.2881, 0.5781)$	$(0.6927, 0.2015)$	0.3550	0.7456	0.2982	0.1193	0.9111	5

Table 7. Sensitivity to the Dombi parameter ξ.

ξ	$\overline{A}_1$	$\overline{A}_2$	$\overline{A}_3$	$\overline{A}_4$	$\overline{A}_5$	$\overline{A}_6$	Ranking
0.5	0.9441	1.0227	1.0078	0.8504	0.9802	0.8938	$\overline{A}_2 \succ \overline{A}_3 \succ \overline{A}_5 \succ \overline{A}_1 \succ \overline{A}_6 \succ \overline{A}_4$
1	0.9462	1.0457	1.0516	0.8691	0.9892	0.9009	$\overline{A}_3 \succ \overline{A}_2 \succ \overline{A}_5 \succ \overline{A}_1 \succ \overline{A}_6 \succ \overline{A}_4$
2	0.9510	1.0775	1.1280	0.8928	1.0048	0.9111	$\overline{A}_3 \succ \overline{A}_2 \succ \overline{A}_5 \succ \overline{A}_1 \succ \overline{A}_6 \succ \overline{A}_4$
3	0.9563	1.0920	1.1642	0.9052	1.0150	0.9180	$\overline{A}_3 \succ \overline{A}_2 \succ \overline{A}_5 \succ \overline{A}_1 \succ \overline{A}_6 \succ \overline{A}_4$
5	0.9668	1.1024	1.1907	0.9162	1.0255	0.9267	$\overline{A}_3 \succ \overline{A}_2 \succ \overline{A}_5 \succ \overline{A}_1 \succ \overline{A}_6 \succ \overline{A}_4$
10	0.9835	1.1091	1.2078	0.9247	1.0336	0.9360	$\overline{A}_3 \succ \overline{A}_2 \succ \overline{A}_5 \succ \overline{A}_1 \succ \overline{A}_6 \succ \overline{A}_4$

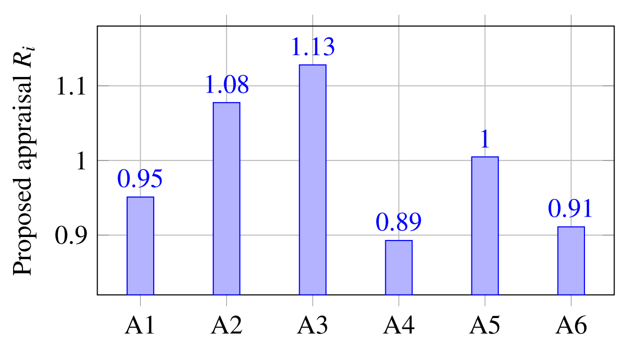

Figure 2. Final IFDA–RANCOM–AROMAN appraisal values.

7.2. *Sensitivity to the two-normalization fusion coefficient*

For the interior range $0.25 \leq \beta \leq 0.90$, the complete ranking is unchanged. At $\beta = 0.10$, only $\overline{A}_1$ and $\overline{A}_6$ exchange positions; the first three alternatives remain unchanged.

7.3. *Sensitivity to the AROMAN criterion-type coefficient*

The baseline order is invariant for $0.2 \leq \lambda \leq 0.8$. The endpoints deliberately suppress one of the two utility components and therefore produce different rankings. This confirms that an interior value should be used when both criterion types are substantively relevant.

7.4. *Sensitivity to RANCOM weights*

Every criterion weight is independently perturbed by $\pm 15\%$ and the vector is renormalized. Table 10 shows that the complete ranking remains unchanged in all ten scenarios.

7.5. *Comparative analysis*

The same criterion weights and the same benefit-equivalent information are supplied to direct IFDA aggregation, TOPSIS, VIKOR, CoCoSo, and a scalar RANCOM–AROMAN implementation [15]. Spearman and Kendall correlations are calculated against the proposed ranking.

The scalar RANCOM–AROMAN calculation produces the same order as the proposed framework, which supports the numerical plausibility of the result. The proposed model nevertheless retains membership, non-membership, and hesitation through normalization fusion and utility construction. CoCoSo and TOPSIS also keep $\overline{A}_2$, $\overline{A}_3$, and $\overline{A}_5$ in the leading group, although their internal order differs. These differences are expected because the methods embody different compensation and distance principles.

7.6. *Methodological implications and limitations*

The proposed framework gives the Dombi–Archimedean algebra an operational role at each uncertainty-sensitive stage. The parameter ξ controls nonlinear compensation, β controls the relative influence of the two AROMAN normalizations, and λ controls the balance between the benefit and cost-desirability components. Reporting all three parameters makes the decision attitude transparent.

The numerical example remains hypothetical. Consequently, the ranking should not be interpreted as a recommendation for any identifiable commercial platform. A submission based on a real procurement problem should replace (84) with independently collected expert assessments, preregister criterion definitions, document conflicts of interest, and validate the ranking against implementation outcomes.

Table 8. Sensitivity to the normalization-fusion coefficient β.

β	$\overline{A}_1$	$\overline{A}_2$	$\overline{A}_3$	$\overline{A}_4$	$\overline{A}_5$	$\overline{A}_6$	Ranking
0.10	0.9342	1.0279	1.0362	0.9267	0.9738	0.9387	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_6 > \overline{A}_1 > \overline{A}_4$
0.25	0.9413	1.0613	1.0885	0.9160	0.9939	0.9298	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
0.50	0.9510	1.0775	1.1280	0.8928	1.0048	0.9111	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
0.75	0.9589	1.0827	1.1491	0.8581	1.0089	0.8843	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
0.90	0.9630	1.0841	1.1579	0.8254	1.0102	0.8608	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$

Table 9. Sensitivity to the AROMAN coefficient λ.

λ	$\overline{A}_1$	$\overline{A}_2$	$\overline{A}_3$	$\overline{A}_4$	$\overline{A}_5$	$\overline{A}_6$	Ranking
0.0	1.4102	1.6520	1.6031	1.2932	1.6484	1.2982	$\overline{A}_2 > \overline{A}_5 > \overline{A}_3 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
0.2	1.0945	1.2615	1.2915	1.0181	1.1905	1.0335	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
0.4	0.9510	1.0775	1.1280	0.8928	1.0048	0.9111	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
0.6	0.9208	1.0102	1.0601	0.8784	0.9539	0.8956	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
0.8	0.9701	1.0103	1.0556	0.9537	0.9716	0.9676	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
1.0	1.0806	1.0509	1.0948	1.1102	1.0264	1.1193	$\overline{A}_6 > \overline{A}_4 > \overline{A}_3 > \overline{A}_1 > \overline{A}_2 > \overline{A}_5$

Table 10. Single-weight perturbation of the IFDA–RANCOM vector.

Scenario	ω_1	ω_2	ω_3	ω_4	ω_5	Ranking
$\overline{C}_1$ -15%	0.3235	0.2960	0.2114	0.1268	0.0423	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
$\overline{C}_1$ +15%	0.3928	0.2657	0.1898	0.1139	0.0380	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
$\overline{C}_2$ -15%	0.3758	0.2484	0.2088	0.1253	0.0418	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
$\overline{C}_2$ +15%	0.3455	0.3090	0.1919	0.1152	0.0384	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
$\overline{C}_3$ -15%	0.3711	0.2887	0.1753	0.1237	0.0412	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
$\overline{C}_3$ +15%	0.3495	0.2718	0.2233	0.1165	0.0388	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
$\overline{C}_4$ -15%	0.3666	0.2851	0.2037	0.1039	0.0407	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
$\overline{C}_4$ +15%	0.3536	0.2750	0.1965	0.1356	0.0393	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
$\overline{C}_5$ -15%	0.3622	0.2817	0.2012	0.1207	0.0342	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
$\overline{C}_5$ +15%	0.3579	0.2783	0.1988	0.1193	0.0457	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$

Table 11. Comparative ranks and rank correlations.

Method	A1	A2	A3	A4	A5	A6	Spearman	Kendall	Ranking
Proposed IFDA–RANCOM–AROMAN	4	2	1	6	3	5	1.0000	1.0000	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
IFDA–RANCOM direct aggregation	6	1	3	5	2	4	0.6571	0.4667	$\overline{A}_2 > \overline{A}_5 > \overline{A}_3 > \overline{A}_6 > \overline{A}_4 > \overline{A}_1$
RANCOM–TOPSIS	4	2	3	6	1	5	0.7714	0.6000	$\overline{A}_5 > \overline{A}_2 > \overline{A}_3 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
RANCOM–VIKOR	3	2	4	6	1	5	0.6000	0.4667	$\overline{A}_5 > \overline{A}_2 > \overline{A}_1 > \overline{A}_3 > \overline{A}_6 > \overline{A}_4$
RANCOM–CoCoSo	4	1	3	6	2	5	0.8286	0.7333	$\overline{A}_2 > \overline{A}_5 > \overline{A}_3 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$
RANCOM–AROMAN	4	2	1	6	3	5	1.0000	1.0000	$\overline{A}_3 > \overline{A}_2 > \overline{A}_5 > \overline{A}_1 > \overline{A}_6 > \overline{A}_4$

8. Conclusion

This study developed an integrated IFDA–RANCOM–AROMAN framework for multi-criteria decision-making under intuitionistic fuzzy information. The principal methodological contribution lies in retaining the intuitionistic fuzzy Dombi–Archimedean mechanism throughout the major uncertainty-sensitive stages of the decision process rather than restricting fuzzy aggregation to a single preliminary step. The proposed IFDA–RANCOM procedure aggregates uncertain criterion-importance assessments, determines criterion priorities, constructs the RANCOM relation matrix, and generates a normalized criterion-weight vector. Subsequently, the IFDA–AROMAN procedure integrates linear and vector normalization, intuitionistic fuzzy embedding, multi-expert aggregation, Dombi–Archimedean normalization fusion, cost complementation, benefit–cost utility construction, and final alternative appraisal. The mathematical consistency of the framework is supported through endpoint-complete Dombi operations and aggregation formulations satisfying the required intuitionistic fuzzy feasibility conditions.

The proposed framework was demonstrated through the selection of six AI-enabled telemedicine platforms for rural healthcare. The IFDA–RANCOM stage produced the criterion-weight vector

$$\omega = (0.36, 0.28, 0.20, 0.12, 0.04),$$

while, at the baseline parameter setting $\xi = 2$, $\beta = 0.5$, and $\lambda = 0.4$, the IFDA–AROMAN procedure generated the ranking

$$\overline{A}_3 \succ \overline{A}_2 \succ \overline{A}_5 \succ \overline{A}_1 \succ \overline{A}_6 \succ \overline{A}_4.$$

Thus, $\overline{A}_3$, representing the mobile-first rural telemedicine platform, emerged as the most preferred illustrative alternative. Comparative analysis showed that the conventional RANCOM–AROMAN approach reproduced the same complete ranking, while other benchmark methods maintained a broadly consistent leading group. Moreover, the leading alternatives remained stable across the examined parameter variations, and the baseline ranking exhibited strong resistance to $\pm 15\%$ single-criterion-weight perturbations.

The results indicate that the proposed IFDA–RANCOM–AROMAN framework provides a transparent, reproducible, and robust mechanism for decision problems involving uncertain expert evaluations and conflicting benefit and cost criteria. Its principal advantage is the systematic preservation of membership, non-membership, and hesitation information across criterion weighting, normalization fusion, utility construction, and final ranking. The present application is illustrative and based on hypothetical data; therefore, the obtained ranking should not be interpreted as a recommendation for any specific commercial telemedicine platform. Future research may extend the framework using real expert and procurement data, interval-valued or type-2 intuitionistic fuzzy information, data-driven expert reliability weights, consensus-based group decision models, stochastic criteria, and validation against actual clinical, economic, and implementation outcomes.

Data availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

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